THE INFLUENCE OF THE LIST ANGLE ON THE SECOND ORDER ROLLING MOMENT OF A LIST SHIP IN RANDOM WAVES

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Abstract

The influence of the transversal list angle on the hydrodynamic rolling moment of waves acting on the ship is investigated by using numerical computations. In the investigation not only the first order wave frequency excitations but also the second order slow oscillation moments are considered. The results show that the list angle may have significant effect on both the average moment and the slow oscillation part of the wave moments. The rolling motion of the ship under the action of a random wave train is calculated by Runge-Kutta integration. The final results are presented in the paper.

1. INTRODUCTION

The motion response of a ship with a list angle in random waves is very interesting in the consideration of ship safety in case the ship is damaged. The motion responses, especially the beam direction motion modes, will have significant effects on the happening of ship capsize. The damaged ship always develops a list angle, which will worsen the stability of the ship and make the ship capsize more likely to happen. It is known that the list angle will have effect on the ship stability curve, which will have some influence on the roll motion. But the hydrodynamic influence of such list angle on the ship motion both first order and second order is not very clear. In order to find the effect of list angle on the hydrodynamic moment acted of roll, a numerical calculation has been carried out for a ship with heavily list angle. The calculation shows that the second order moment of roll increase significantly. Although from the magnitude of the moment increase it seems the list angle will have significant effect on the rolling of ship, the time domain motion calculation gives insignificant increase of roll amplitude. So it gives us some confidence that in the motion prediction of a list ship ignoring the second order effect may be acceptable.

2. CALCULATION PROCEDURE AND THE SECOND ORDER MOMENT

The calculation is performed on a routine program, which uses the frequency domain finite boundary element method. The program can handle a variety of floating body forms, and gives the first and second order hydrodynamic parameters, such as the added mass, damping coefficients and wave excitation forces. In the second order calculation, the second order drifting moment and the frequency function for defining the slow oscillation wave moments are obtained. As for drifting moment, that is

$$\overline{F}_4^{(2)}(\omega) = k(\omega)\rho g a^2 \tag{1}$$

where $\overline{F}_4^{(2)}(\omega)$ is the drift rolling moment, $k(\omega)$ is the drifting moment coefficient as a function of frequency ω. The slow oscillation moment is defined as

$$F^{(2)}(t) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j P_{i,j}(\omega_i,\omega_j)\cos\{(\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)\} + \sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j Q_{i,j}(\omega_i,\omega_j)\sin\{(\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)\} \tag{2}$$

in which $F^{(2)}(t)$ is the slow oscillation wave moment (force) induced by random waves, a_i the amplitude of ith wave component, ω_i the frequency of i th wave component, while ε_i is the ith phase angle, and the functions $P_{i,j}(\omega_i,\omega_j)$, $Q_{i,j}(\omega_i,\omega_j)$ are in-phase and out-of phase second order slow oscillation functions in defining the second order moments.

All of such functions can be obtained by running the program mentioned above. And by using these functions, the time traces of the rolling moment both first and second order can be determined.

For the determination of the second order wave moment acting on a ship in the given random waves, firstly we have to calculate the wave time trace of enough length. Therefore the random wave train is generated, which can be expressed as follows

$$\zeta(t) = \sum_{i=1}^{N} a_i \sin(\omega_i t + \varepsilon_i) \tag{3}$$

where $\zeta(t)$ is the generated wave train or time history, a_i the i th component wave amplitude, ω_i the frequency of i th component wave. ε_i is a random phase angle attributed to the ith wave component. The Probability Distribution Function (PDF) of such random phase is a uniform distribution in the range $(-\pi,\pi)$ with the PDF $1/2\pi$. Due to the inclusion of such random term, it makes the model random. Generally, the amplitude of the component waves are determined by some predetermined wave spectra $S(\omega)$. That is

$$\frac{1}{2}a_i^2 = S(\omega_i)d\omega_i \tag{4}$$

or

$$a_i = \sqrt{2S(\omega_i)d\omega_i} \tag{5}$$

The wave spectra represents the distribution of wave energy along its frequency range, which depends on the wind speed, fetch. As usual, some standard types of spectra are employed to determine the wave amplitudes. Here, in our calculation the following Jonswap spectra is used:

$$S(\omega) = 0.862\frac{0.0135g^2}{\omega^5}exp\left(-\frac{5.186}{\omega^4 h_{1/3}^2}\right)1.63^p \tag{6}$$

where $h_{1/3}$ is the significant wave height, and

$$p = exp\left(-\frac{(\omega-\omega_0)^2}{2\sigma^2\omega_m^2}\right)$$

$$\omega_m = 0.32 + \frac{1.80}{h_{1/3}+0.6}$$

$$\sigma = \begin{cases} 0.08 & if \quad \omega < \omega_m \\ 0.10 & if \quad \omega > \omega_m \end{cases}$$

To determine the wave drifting moment of roll, only the wave frequency second order drift moment coefficients are involved in. The drifting moment does not vary with time, so it does not influence the oscillation motions of a ship. The magnitude of the second order drifting moment in the given random waves can be calculated as follows

$$\overline{F}_4^{(2)} = \int_{-\infty}^{\infty}\rho g k(\omega)S(\omega)d\omega \tag{7}$$

The slow oscillation moment is related to the second order frequency response function or quadratic transfer function $P_{i,j}(\omega_i, \omega_j)$ and $Q_{i,j}(\omega_i, \omega_j)$, and it can be calculated according to equation (2) if the amplitudes of the component waves a_i are given. Actually, the amplitudes of the wave component are already defined by the spectra given in equation (6) and equation (4).

It should be noticed that since the generated time series of waves are finite, which will be truncated at a length, the parameters, such as the number of frequencies and the increment of frequency must fulfil some requirement. As we know, the largest time interval of the curve of the exciting wave is related closely to the frequency increment, or the frequency difference between the frequencies of adjacent components. The length of time interval $T_{reappear}$ is determined by the following relationship

$$T_{reappear} = \frac{2\pi}{\Delta\omega} \tag{8}$$

This means in order to enlarge the time length of the generated wave time trace, the small frequency increment has to be used. The time trace of the random wave out of the time length $T_{reappear}$ generated by the formula (3) will exhibit repeatedly as the former time trace.

In order to calculate the drift moment and generate the slow oscillating moment, the coefficient $k(\omega)$ and the quadratic transfer functions $P_{i,j}(\omega_i, \omega_j)$, $Q_{i,j}(\omega_i, \omega_j)$ have to be determined firstly. As to these functions, they are obtained completely by run of the program. The rolling moments of a barge type ship with 13 degrees list angle that has the parameters in Table 1, are calculated. The results are shown in Fig.1, in which, the up curve stands for the drifting rolling moment of the ship in an up-righting condition, the other denotes the drifting rolling moment of the ship in the condition with a list angle of 13 degree.

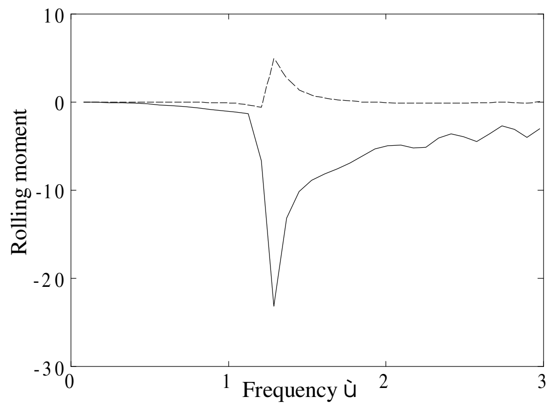

Fig. 1 The drifting roll moment of a barge type ship

The calculated functions $P_{i,j}(\omega_i, \omega_j)$ and $Q_{i,j}(\omega_i, \omega_j)$ are shown in Fig. 2, 3.

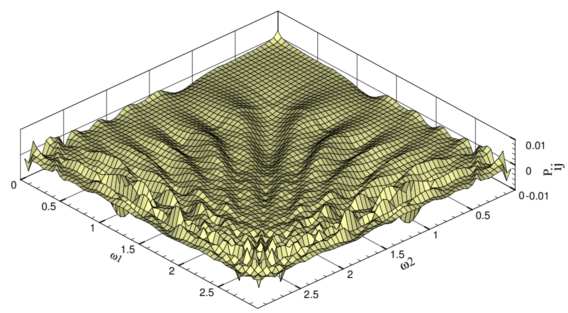

Fig.2 The normalized quadratic transfer functions $P_{i,j}(\omega_i, \omega_j)$

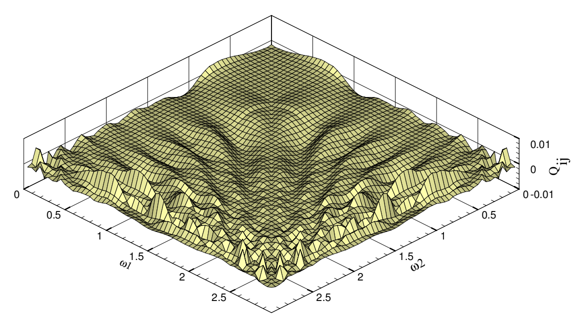

Fig.3 The normalized quadratic transfer functions $Q_{i,j}(\omega_i, \omega_j)$

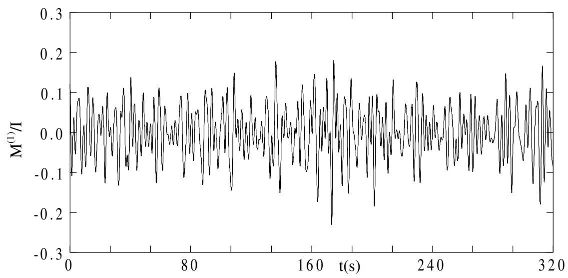

Fig. 4 The normalized moment of the first order wave frequency excitation

The first order wave excitation moment is calculated according to the following considerations,

$$M(t) = I\alpha_0\omega_0^2\pi\sum_{i=1}^{N}\frac{a_i}{\lambda_i}cos(\omega_i t + \varepsilon_i) \tag{9}$$

where I is the total inertia moment of the ship around its longitudinal axis, α_0 is a so called effective steepness coefficient of wave that takes a value of 0.7 here, a_i and λ_i are the wave amplitude and the length of the i th wave component. The normalized moment of the first order wave frequency excitation generated numerically for given wave time trace is shown in Fig.4

The time trace of the second order slow oscillation moments in the given random waves is generated by equation (2), and the normalized moment of the second order slow oscillation is shown in Fig. 5.

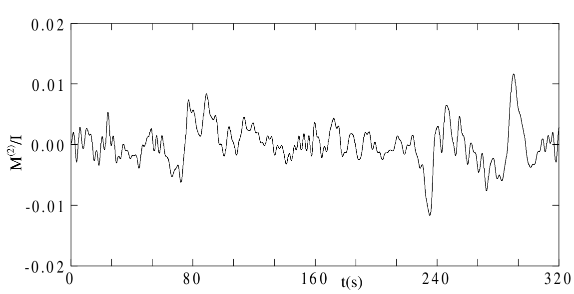

Fig. 5 The normalized moment of the second order slow oscillation

3. THE CALCULATION OF SHIP ON RANDOM WAVES

In order to investigate the significance of the second order roll moment developed by the list angle on the roll motion of a listed ship, the calculation of a listed ship in the given random waves has been carried out. The special thing of a listed ship in the constitution motion equation is that, the existence of the list angle will make the ship stability curve on each side of the roll angle not symmetrical. The rolling motion equation of a listed ship can be depicted as follows

$$I\ddot{\phi} + N\dot{\phi} + C_1\phi - C_2\phi^2 = M(t) \tag{10}$$

in which I is the inertia moment of the ship around its longitudinal axis, N is the damping coefficient, C_1 is the linear restoring coefficient while C_2 is a coefficient of the second order restoring term, which includes the consideration of the non-symmetry of the restoring curve on each side of the list.

4. NUMERICAL EXAMPLE

A barge type ship is employed to investigate the nonlinear rolling motion due to both the first order wave excitation and the second order slow oscillation moments. The principal particulars and parameters of rolling motion of the barge type ship are listed in Table 1.

Table 1 Principal particulars and rolling parameters of the container ship

Items	Ship
Length	L_{pp}=100m
Breadth	B=41m
Depth	D=m
Draught	T=4.8m
Displacement	Ä=16810m^3
Rolling radius	R_{xx}=18.0m
Rolling metrecenter	GM=35.196m
Intrinsic frequency	$\omega_0 = 1.032$ rad/s.

Here only the part of the above mentioned wave spectra is considered that the frequency ù varies in the range of 0~3 rad/s, and the frequency step •ù=0.02rad/s is applied to determine the wave excitation. The given random wave excitation with respect to a series of significant wave heights are generated by Equation (3). By use of the quadratic transfer functions $P_{i,j}(\omega_i,\omega_j)$ and $Q_{i,j}(\omega_i,\omega_j)$ calculated in advance, both the first order and the second order slow oscillation moments are obtained. As shown in the foregoing figures Fig.4 and 5, the normalized moments both the first order wave excitation and the second order slow oscillation are obtained when the significant wave height $h_{1/3}$=4m, and the excitation wave is generated by the summation of the 150 waves, and the initial phases are generated by computer randomly.

As for the nonlinear rolling motion of Equation (8), it was solved by four-order Runge-Kutta integration method. For the length of time interval $T_{reappear}$=314s, the above solution of the roll motion is truncated at 320 seconds. The time step •t=0.05, and the initial condition $\varphi(0)=0$, $\dot{\varphi}(0)=0$. Fig.6 and 7 show the time history of the rolling motion of the ship due to the first order wave excitation and the second order slow oscillation moment when the significant wave height $h_{1/3}$=2m, 4m, 6m respectively.

5. CONCLUDING REMARKS

(1) The second order drifting moment will increase with the increase of list angle. It's already shown in Fig.1.

(2) The second order slow oscillating moment also increases with the increase of list angle.

(3) Although the slow oscillation moment increases in list condition, the motion calculation shows that the rolling motion induced by the second order moment is less than the 10 percent of the total motion.

(4) It seems that we can ignore the influence of the second order moment in the prediction of motion of list ship in random waves.

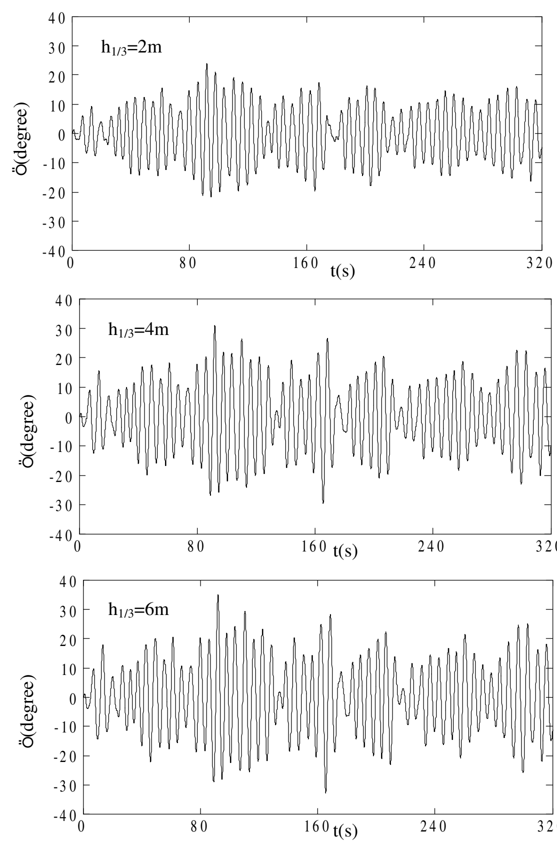

Fig.6 Time histories of rolling motion due to the first order wave excitations

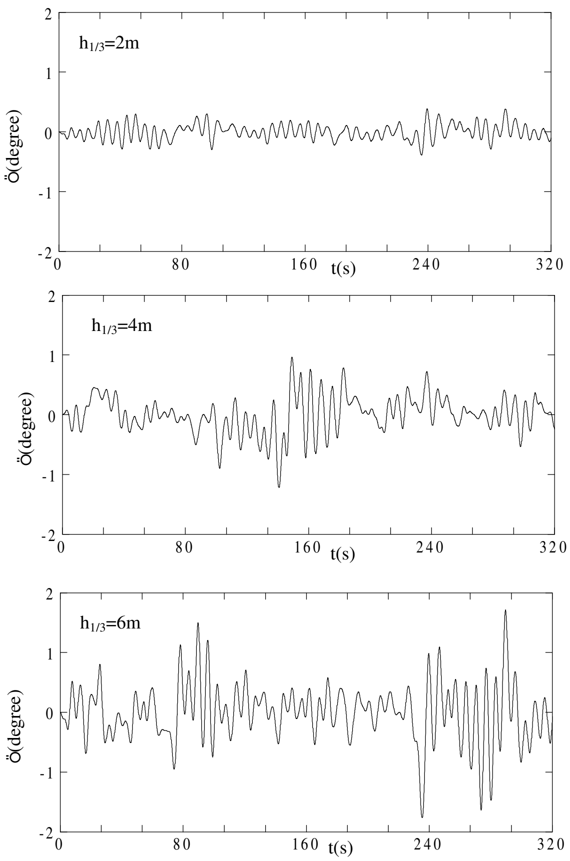

Fig.7 Time histories of rolling motion due to the second order slow oscillation moment

6. REFERENCES

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